5-1 Operations w/ Polynomials

Monomial - a number, variable, or expression that is the product (×) of one or more variables w/ non negative exponents.
ex: 3, x, x^2, x^2y, $2xy^3$

x^a x is the base, a is the exponent or degree.

Properties
- $x^a \cdot x^b \rightarrow x^{a+b}$ — mult 2 monomial w/same base: add exp
- $x^a / x^b \rightarrow x^{a-b}$ — Divide 2 monomial w/same base: sub exp.
- $x^{-a} \rightarrow 1/x^a$
- $(x^a)^b \rightarrow x^{a \cdot b}$ — raise a monomial to a power, mult exp → all parts of the mono get the exponent
- $(x \cdot y)^a \rightarrow x^a y^a$
- $(x/y)^a \rightarrow x^a/y^a$
- $x^0 = 1$ — anything to the power of zero is 1

Simplified Monomial:
- no powers of powers
- each base appears once
- all fractions are in simplest form
- no negative exponents

Simplifying Expressions

① $(2a^{-2})(3a^3b^2)(c^{-2})$

$2 \cdot \frac{1}{a^2} \cdot 3 \cdot a^3 \cdot b^2 \cdot \frac{1}{c^2}$

$2 \cdot 3 \cdot \frac{1}{a^2} \cdot a^3 \cdot b^2 \cdot \frac{1}{c^2}$

$6 \cdot \frac{a^3}{a^2} \cdot b^2 \cdot \frac{1}{c^2}$

$6 \cdot a^{3-2} \cdot b^2 \cdot \frac{1}{c^2}$

$6 \cdot a \cdot b^2 \cdot \frac{1}{c^2} = \frac{6ab^2}{c^2}$

① get rid of negative exponents
② bring together like terms
③ mult like terms
④ simplify

② $\frac{q^2r^4}{q^7r^3}$

$q^{2-7} \cdot r^{4-3}$

$q^{-5} \cdot r^1$

$\frac{r^1}{q^5}$

① use division property
② simplify
③ write w/ positive exponents

What is a polynomial?
an expression consisting of monomials that are being added or subtracted.

- no square roots of variables
- no fractional or negative powers on the variables
- no variables in the denominator

Determine if each expression is a polynomial, if so, state the degree (largest exponent of the poly)

① $3x^3 + 4x^5 + 3$ yes, degree = 5

② $\sqrt{x} + x + 4$ No, sqrt

③ $x^{-2} + x - 3$ No, neg exp

Simplify

① $(4x^2 - 5x + 6) - (2x^2 + 3x - 1)$

$4x^2 - 5x + 6 - 2x^2 - 3x + 1$

$4x^2 - 2x^2 - 5x - 3x + 6 + 1$

$2x^2 - 8x + 7$

① remove parenthesis (apply neg to second poly)
② combine like terms
③ simplify

Simplify

② $(6x^2-7x+8)+(-4x^2+9x-5)$

$6x^2-7x+8+-4x^2+9x-5$

$6x^2-4x^2-7x+9x+8-5$

$2x^2+2x+3$

① remove parentheses
② combine like terms
③ simplify

③ $3x(2x^2-4x+6)$

$3x(2x^2)+3x(-4x)+3x(6)$

$6x^3-12x^2+18x$

① Distribute the $3x$
② mult. like terms
③ no like terms to combine

④ $-2a(-3a^2-11a+20)$

$-2a(-3a^2)-2a(-11a)-2a(20)$

$6a^3+22a^2-40a$

⑤ $(n^2+4n+6)(n+2)$

$(n^2+4n-6)(n+2)$

$n^3+2n^2+4n^2+8n-6n-12$

$n^3+6n^2+2n-12$

① multiply (similar to FOIL)
② combine like terms

Algebra 2 Name_____

5-1 Practice Problems

Simplify. Your answer should contain only positive exponents.

1) $3x \cdot 3x^2y^4$

2) $4a^2b^{-2} \cdot ab^3$

3) $3v^{-4} \cdot v^3$

4) $yx^{-3} \cdot yx^4$

5) $4ab^4 \cdot 3a^{-1}b^2$

6) $2x^3 \cdot 4y$

7) $(b^2)^2$

8) $(2x^2y^4)^{-1}$

9) $(3x^{-3}y^4)^3$

10) $(2mn^2)^4$

11) $(3ab^4)^3$

12) $(m^2n^3)^3$

13) $\dfrac{3a^{-3}b^4}{b}$

14) $\dfrac{x^4y^2}{4x^{-3}}$

15) $\dfrac{4xy^{-2}}{4xy^3}$

16) $\dfrac{x^2}{x^4y^3}$

17) $\dfrac{2a^3}{4a^{-3}b^{-3}}$

18) $\dfrac{2yx^{-3}}{2x^2y^2}$

Simplify. Your answer should contain only positive exponents. (These are tough)

19) $\dfrac{(2x^{-2})^{-2}}{x^2y^{-1}\cdot 2y^{-1}}$

20) $\dfrac{(2x^{-3})^{3}}{2x^{-4}y^{3}\cdot 2x^{-1}y^{3}}$

21) $\dfrac{2u^4v^4\cdot(2u^4)^2}{(u^4v^4)^4}$

22) $\dfrac{(2x^4)^{-1}}{yx^3\cdot y}$

23) $\dfrac{(nm^{-1}\cdot m^{-1}n^3)^4}{2m^2n^{-4}}$

24) $\dfrac{(ba^2)^4}{a^3\cdot 2b^3}$

25) $\left(\dfrac{2x^{-3}y^2\cdot x^3y^2}{x^3y^{-4}}\right)^2$

26) $\left(\dfrac{x^{-2}y^3}{x\cdot 2x^4}\right)^4$

Simplify each expression.

27) $(8m^4-6+4m)+(7m^3+7m^4-1)$

28) $(4x^3+x^2+4x^4)+(2x^3-3x^2-4x^4)$

29) $(5v+2-8v^3)+(5-4v^3-5v)$

30) $(3n-2n^2+4)+(4n^3-6n^2+6)$

31) $(3a-a^2+6)+(a^2-a-3)$

32) $(6a^2+2a^3+1)+(3a^2-5a^3+7)$

Find each product.

33) $(2n+6)(3n^2+n+7)$

34) $(a+1)(5a^2+4a+6)$

35) $(5k-6)(k^2+3k-5)$

36) $(6x-7)(4x^2-6x-4)$

37) $(2r+5)(r^2-8r+1)$

38) $(p+7)(p^2+4p+2)$

Answers to 5-1 Practice Problems

1) $9x^3y^4$

2) $4a^3b$

3) $\frac{3}{v}$

4) y^2x

5) $12b^6$

6) $8x^3y$

7) b^4

8) $\frac{1}{2x^2y^4}$

9) $\frac{27y^{12}}{x^9}$

10) $16m^4n^8$

11) $27a^3b^{12}$

12) m^6n^9

13) $\frac{3b^3}{a^3}$

14) $\frac{x^7y^2}{4}$

15) $\frac{1}{y^5}$

16) $\frac{1}{x^2y^3}$

17) $\frac{a^6b^3}{2}$

18) $\frac{1}{x^5y}$

19) $\frac{x^2y^2}{8}$

20) $\frac{2}{x^4y^6}$

21) $\frac{8}{u^4v^{12}}$

22) $\frac{1}{2x^7y^2}$

23) $\frac{n^{20}}{2m^{10}}$

24) $\frac{ba^5}{2}$

25) $\frac{4y^{16}}{x^6}$

26) $\frac{y^{12}}{16x^{28}}$

27) $15m^4 + 7m^3 + 4m - 7$

28) $6x^3 - 2x^2$

29) $-12v^3 + 7$

30) $4n^3 - 8n^2 + 3n + 10$

31) $2a + 3$

32) $-3a^3 + 9a^2 + 8$

33) $6n^3 + 20n^2 + 20n + 42$

34) $5a^3 + 9a^2 + 10a + 6$

35) $5k^3 + 9k^2 - 43k + 30$

36) $24x^3 - 64x^2 + 18x + 28$

37) $2r^3 - 11r^2 - 38r + 5$

38) $p^3 + 11p^2 + 30p + 14$